MathPASS: A Remedial Mathematics System with Automated Answer Checking

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Abstract

Introduced is a Web-based intelligent mathematics assessment system, MathPASS. MathPASS can generate random mathematical test questions based on pre-determined question prototypes, let users enter their answers in either a WYSIWYG or a character-string input environment, and check user answers with the help of a computer algebra system (CAS).

1 MathPASS Overview

The application of computer technology in education [11] has become increasingly important as we move into the information age. In particular, a computer aided assessment (CAA) system [1, 4, 5, 6, 7, 9] can be very useful for teachers and students. We introduce a Web-based intelligent mathematics assessment system, MathPASS [16], developed jointly at Lanzhou University (LZU) and Kent State University (KSU). MathPASS is a drill-and-practice system which has been applied in remedial mathematics courses at KSU. Four types of users, guest, student, teacher, and administrator, have different privileges for using MathPASS. Figure 1 shows a screen shot of `Take Assignment` page for student user in MathPASS. By June 2010, in MathPASS users include 5900 students and 100 teachers, taken assignments are more than 100 thousands, and taken questions are more than 1 million. MathPASS can be accessed at http://mathpass.math.kent.edu.

As a practical system, MathPASS provides more than 10 modules including: class management, assignment management, grade management, account management, student management, result analysis and etc. MathPASS can generate random mathematical test questions based on pre-determined question prototypes, let users enter their answers in either a WYSIWYG or a character-string input environment, and check user answers with the help of a computer algebra system (CAS) [12]. Figure 2 shows the main architecture of MathPASS.

More than 400 question prototypes have been created in MathPASS by the combined efforts of mathematics teachers and mathematics education experts at KSU. A question prototype is a mathematical question template for a specific question pattern containing random parameters satisfying well-defined mathematical conditions. The different question instances of one question prototype are generated on-the-fly for each test, quiz, or homework. A guiding principle in the design of question prototypes is to split a test topic into a range of separate cases constituting significantly different experiences for a learner whose development in that topic is still at an early stage. Table 1 lists 5 question prototypes which are in the field of exponent manipulation. The table also gives the statistic result including taken times, correct times, and the correct ratio of the question. All of the statistic data can be accessed by a MathPASS teacher user who may use these data in preparing his/her course.

MathPASS employs MathEdit [13], a Web-based mathematical expression editor, to let users enter answers. With MathEdit, students can enter their answers via WYSIWYG editing or command-line editing mode in MathPASS. In WYSIWYG editing mode, students can create and edit mathematical expressions with a convenient and intuitive graphical user interface (GUI). In command-line editing mode, students can enter mathematical expressions with character-string input.

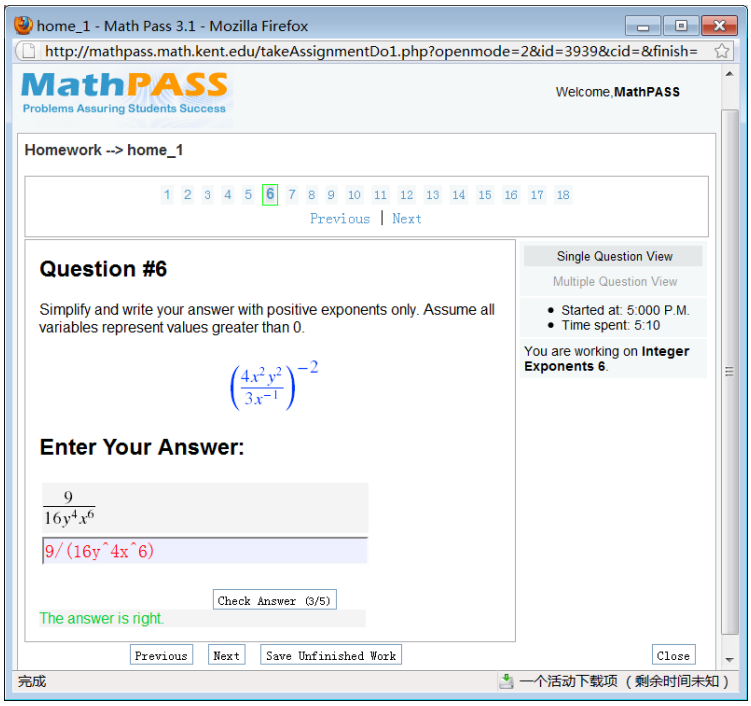

Figure 1: A screen shot of MathPASS

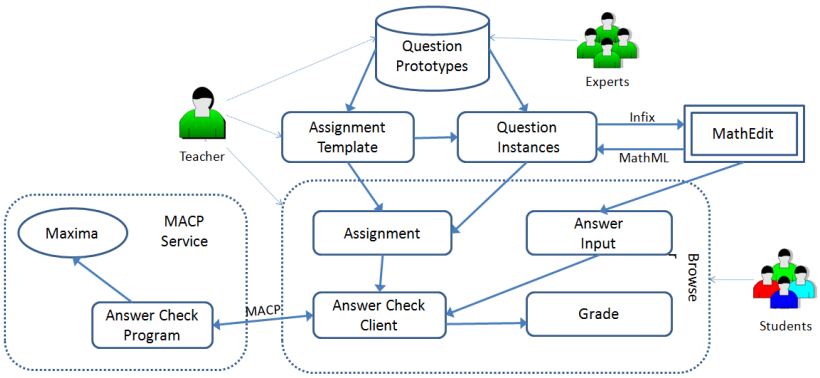

Figure 2: MathPASS System Architecture

Table 1: Question Prototypes of MathPASS

Description	Prototype	Instance	Taken Times	Correct Times	Percentage
Exponent Manipulation 1	$\frac{x^A y^B}{(x^C y)^D}$	$\frac{x^{-2}y^{-3}}{(x^4y)^2}$	2004	1001	49.95%
Exponent Manipulation 2	$\frac{(x^A y)^B}{(x^C y)^D}$	$\frac{(x^4y)^{-3}}{(x^{-5}y)^2}$	1971	947	48.05%
Exponent Manipulation 3	$(\frac{x^A y^B}{z^C})^D$	$(\frac{x^{-5}y^2}{z^5})^4$	1909	975	51.07%
Exponent Manipulation 4	$(\frac{Ax^C y^D}{Bx^E})^F$	$(\frac{5x^4y^8}{4y^2})^{-3}$	1908	742	38.89%
Exponent Manipulation 5	$(\frac{Ax^C y^D}{Bx^E y^F})^G$	$(\frac{2x^5y^{-6}}{3x^{-2}y^3})^{-3}$	1872	723	38.62%

2 MACP

In recent years some computer aided assessments of mathematics employ computer algebra system (CAS) to evaluate the work of students [5, 6, 10]. As a part of MathPASS, LZU and KSU are jointly designing and developing a mathematics answer checking protocol (MACP). MACP is an access protocol for communication between MACP service and its client. MACP uses HTTP as its basic protocol and uses JSON as its data transfer format. The following shows an example of request data in MACP:

```
[{
"name":"c1",
"cmd":"(u1=s1&u2=s2)|(u1=s2&u2=s1)",
"request":["ranme":"u1.mmlp","rname":"u2.mmlp"],
"exprs":[
  {"expr":"s1","encoding":"mmlc","role":"stdAnswer",
   "value":"<math><plus/><apply><minus/><cn>2</cn></apply><apply><divide/>
   <apply><root/><cn>2</cn></apply><cn >2</cn></apply></math>"},
  {"expr":"s2","encoding":"mmlc","role":"stdAnswer",
   "value":"<math><minus/><apply><minus/><cn>2</cn></apply><apply><divide/>
   <apply><root/><cn>2</cn></apply><cn >2</cn></apply></math>"},
  {"expr":"u1","encoding":"infix","role":"usrAnswer","value":"(-4-sqrt(2))/2"},
  {"expr":"u2","encoding":"infix","role":"usrAnswer","value":"sqrt(8)/2-2"}
 ]
}]
```

In the example, *u1* and *u2* are two user answers and *s1* and *s2* are two standard answers for the question of *solving the equation* $2x^2 + 8x + 7 = 0$. The answer checking rule is to judge whether *s1=u1* and *s2=u2* or *s1=u2* and *s2=u1*. The return result of MACP is shown as the following:

```
[{
  "name":"c1",
  "status":"normal",
  "correctness":"96%",
  "details":"<u2> is not simplest",
  "response":[
    {"rname":"u1.mmlp",
     "value":"<math><mfrac><mrow><mo>-</mo><mn>4</mn><mo>-</mo>
     <msqrt><mn>2</mn></msqrt></mrow><mn>2</mn></mfrac></math>"},
    {"rname":"u2.mmlp",
     "value":"<math><mfrac><msqrt><mn>8</mn></msqrt><mn>2</mn></mfrac><mo>-</mo><mn>2</mn></math>"
    }]
}]
```

The value of the `correctness` and `details` parameter denotes that the user answer gets a score of 96% and the answer *u2* is not in simplest form. The MathML Presentation [15] code of *u1* and *u2* is also returned as a request-response item.

MACP service is a Web service for checking answers and grading questions to mathematics of middle-school and college. The MACP service aims to grade user answers through verifying the equivalence of expressions and checking expression forms of the answers. The implementation of MACP is based on Representational State Transfer (REST). When a MACP request is received, the MACP service parses the logic expressions given in the *cmd* parameter and verify the equivalence of the corresponding expressions via a server-side computer algebra system, Maxima. The standard answers could also be computed via Maxima. MACP service also provides the conversion service for different mathematical expression formats including: MathML Content, MathML Presentation, OpenMath [14], LaTex and picture (see the figure 3).

With the analysis of correct answer or best answer, the authors find that most of them in CAA systems could be classified into two broad categories: simplest form and special syntax form. The service may also judge whether

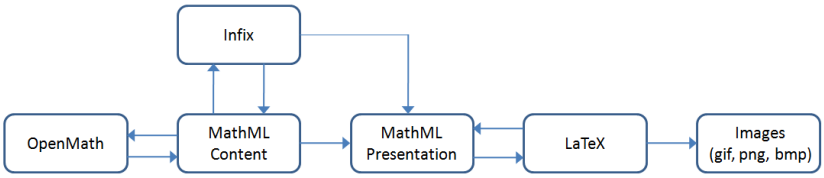

Figure 3: The conversion service for mathematical expressions

a student answer is in a simplest or a required form. This taxonomy does not separate strictly all the answers into two totally different categories. One part of the answer with special syntax form may also be required as a simplest form. However this taxonomy has been found to be useful for evaluating the answer of students and solving the issues of partial credit.

The first category of correct answer is to ask the student to enter the answer with a full simplification form. The question of this category included simplification, calculation, writing the equation of a function, writing the domain of expression, and etc. Particularly students are desired to reduce their answers to lowest terms. To verify an expression in simplest form, MACS will rewrite the student's expression in a canonical form. The meaning of canonical form in the paper is different with the one in some other literatures such as [2, 3, 8]. The later represents an alternative to the notion of simplified form. The purpose of our canonical form of an expression is to make it easy to verify it in simplest form. So we only use some rewriting rules such as commutative law and associative law of addition and multiplication to convert an expression into a specifying pattern. In other words we do not simplify the expression during the canonical process and the length and complexity of the expression were not changed during the rewriting. After getting the canonical forms, MACS will judge whether the student's answer is in simplest form.

The second category of an acceptable correct answer is to ask students to write the answer expression with a special syntax form. The questions of this category include factoring, solving equation, rewriting the expression as request, find the function as request, and etc. Students need write the answer expression as an appointed syntax form such as multiplication of polynomials, a sum or difference of logarithms. This kind of answer does not need to be reduced to lowest terms. Contrarily, the question may be lowest terms and obviously the answer should not be same with the question. More than 30 Pre-defined special forms such as `Simplest Fraction (SF)`, `Simplest Integer (SI)`, `Factoring Polynomial (FP)`, `Normalized Scientific Notation (NSN)`, `Sum and Difference of Logarithm (SDL)`, and `Positive Exponent (PE)` have been defined in MACP service. The user may use logical combination of these special form as a request form. For example a client may send `SF|SI` to statement the answer could be a simplest fraction or a simplest integer. In this case a decimal will be treated as wrong answer.

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