Features and Advantages of WME:
A Web-based Mathematics Education System

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Abstract

An ongoing project creates a Web-based Mathematics Education (WME) system by an innovative combination of standard Web technologies. WME delivers classroom-ready lessons that are well-prepared, interesting, effective, as well as interoperable. Lesson pages contain easy to use special-purpose manipulatives to help students understand and explore mathematical concepts and skills through hands-on activities. Manipulatives are parameterized for easy customization and object-oriented for interoperability. Mathematics-oriented chat and bulletin boards facilitate communication and discussion. Additional features enable teachers to easily tailor lessons to suit the needs of their classes, to create and conduct assessments, and to help students overcome difficulties. A pilot project puts WME through in-class trials with a prototype site, a step towards a model WME site that can be easily deployed to different schools. WME has many advantages and the potential to create a Web for mathematics education, to foster a new paradigm for supporting and delivering mathematics education, and to help mathematics curricula improve exponentially.

1. Motivation

There is much research in mathematics education that indicates students in the US are falling behind other countries in the world and that gaps between low SES and minority students are increasing [3]. In addition, distinguished groups of scientists and educators have reviewed research and organized recommendations for mathematics and science education, which have been for the most part ignored [29]. The type of mathematics that is emphasized in this country has not adapted to the changing requirements of the job market. According to the National Research Council [31], "The fastest growing jobs require much higher math, language, and reasoning abilities than current jobs, while slowly growing jobs require less." All this research and the many reports generated over the past 15 years clearly state that students need to master mathematics more deeply and at higher levels than they do at present time.

The increased curricular demands on students means mathematics teachers are required to know more mathematics themselves. This coupled with the push for high stakes tests has caused shortages in highly qualified mathematics teachers. The amount of cognitive, organizational, and emotional work needed to be a successful mathematics teacher, already high, is growing. What can be done to help mathematics teachers do their job more effectively for more students?

Considered here is a Web-based Mathematics Education (WME) system to provide students with high quality interactive lessons as well as interesting and relevant contexts to increase motivation for learning mathematics. WME can help teachers perform research-based tasks and assess student performance to obtain formative and summative feedback. WME can also supply an online communication system for students and teachers to openly interact and discuss mathematical ideas inside or outside the classroom.

In a summary of research on Technology and Mathematics Education [12], Kaput indicates that a wide variety of research studies, from calculators to computer symbolic computation and graphic-drawing programs, have shown no negative effects on learning basic mathematical ideas by students. Furthermore, the potential of computers to help students learn more and more significant mathematics is limited only by availability of computers for classroom use, interactive programs available for use, and a cohesive curriculum that has sufficient flexibility to foster genuine inquiry by students (p. 548).

More recent studies involving an interactive program called Logo [8] indicate that when the use of computers is guided by knowledge of how students learn specific mathematics topics, students learned more and gained a deeper understanding of mathematics. "...it has been shown that middle school students using Logo move to higher levels of conceptualizing and begin to integrate spatial and symbolic mental representations." If teachers have access to a system that provides a rich set of ready-to-use interactive lessons, guide for concrete tasks to teach required mathematics topics, and an assessment system to identify student needs and to choose follow-on lessons, they will be able to help their students learn more and gain a deeper understanding. Through the Web, WME can enable many teachers to help students meet standards recommended by the National Council of Teachers of Mathematics (NCTM) [30] for mathematics education. Because its lessons, manipulatives and assessments can be customized and shared online easily among educators and researchers, WME has the potential to create a new paradigm for

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supporting and delivering mathematics education and to help mathematics curricula improve exponentially.

2. Introduction and Background

The NSF Networking for Education Testbed (NET) program is an early project that fostered the use of the Internet in schools [17]. But, effective use of technology in schools remains a challenge, especially for science and mathematics [9]. Investigators at the W3 Consortium (W3C) and elsewhere are working to make publishing mathematical materials on the Web easy. MathML [27] is an XML application for markup of mathematical expressions that supports both presentation encoding (display layout) and content encoding (computation semantics).

Popular Web browsers such as Netscape, FireFox and Mozilla are beginning to support MathML display natively while the Microsoft Internet Explorer relies on plug-ins, such as the free MathPlayer from Design Science Inc., to display MathML. The increasing acceptance and software support for MathML were evident at recent *MathML International Conferences* [28].

Scalable Vector Graphics (SVG) [10] is another developing standard of W3C. SVG is also an XML application that allows easy coding of vector graphics for geometric objects and any transformation on them defined by a coordinate transform matrix. Animations conforming to SMIL [2] are also supported. SVG is a very useful language with which to build Web-based interactive tools for graphing, plotting, and geometry.

Traditionally teachers have relied on hardcopy publications, such as Motivation Counts [11], *Teaching Secondary Mathematics: Techniques and Enrichment Units* [15], and *101 Great Ideas for Introducing Key Concepts in Mathematics: A Resource for Secondary School Teachers* [16], for ideas and methods. Researchers and companies have begun to provide various types of mathematics education materials over the Web. For example:

- The Ohio Resource Center for Mathematics, Science, and Reading provides online resources for mathematics education [32].
- The United States Department of Education site (`www.ed.gov`) has a *Mathematics* section that provides a vast amount of useful tools and information.
- The IES sponsored Education Resources Information Center [22] offers Web access to an extensive database of journal and non-journal education literature.
- The Eisenhower National Clearinghouse for Mathematics and Science Education (ENC) [23] links to lesson plans and activities in many topic areas.
- The NCTM (`illustrations.nctm.org`) Illuminations Project supplies a set of self-contained applets for illustrating concepts and ideas at different grade levels. NCTM also provides lessons developed by teachers.
- The Public Broadcasting System Mathline site (`www.mpt.org/learningworks/teachers/mathline`) provides lesson plans and activities targeted to specific grade levels.
- The National Library of Virtual Manipulatives for Interactive Mathematics (`matti.usu.edu/nlvm/nav`) at Utah State provides a variety of applets for interactive manipulation by students to learn different mathematical concepts.
- Mathforum (`mathforum.org`) at Drexel University provides *Problem of the Week* and *Mathforum Math Library* among other useful materials.
- Other websites provide resources, courses and tools for mathematics education, such as Livemath, WIMS, Internet4Classrooms, WebMathematica, Mathwright, geometry.net, Calc101, AcitveMath, Maple, and MathWeb. Linda Beccerra [4] gave a good summary of on-Web tools for interactive computation.
- General e-learning and e-education support infrastructure systems such as WebCT and Blackboard (`www.blackboard.com`) provide campus-wide or district-wide course organization, user tracking, message boards, content delivery, administrative control, parent involvement, and other services.

2.1. WME is Different

WME is different as compared to these existing approaches and can provide several significant advantages:

- Generally, a stand-alone applet, virtual manipulative, or server-side program supports a narrow set of topics or operations. They are hard to use by teachers in the classroom. Building lesson pages to incorporate such tools requires non-trivial amounts of time, effort, and programming expertise. Further, these tools are unrelated and do not interoperate to mutually reinforce. WME offers classroom-ready lessons and modules that are complete and interoperable among all WME sites, therefore maximizing convenience and usability.
- Topic and lesson resources help teachers retrieve useful teaching information from the Web. But these are not a substitute for complete Web-based curricula, built by experts and conforming to standards, for entire semesters and grade levels. WME can deliver such curricula in lesson pages and modules that teachers can customize, modify, and enhance. With WME, teachers have an easy and effective way to deliver ready-made lessons in a tailored form to their classes.

- WME is not a general e-learning infrastructure. It focuses on mathematics education and integrates lessons, manipulatives, assessment tools, and teacher-student interaction for effective teaching and learning of mathematics. WME can be used independently or within a general e-education infrastructure.

2.2. The Beginning of WME

Our group at the Institute for Computational Mathematics (ICM/Kent) has been conducting NSF-sponsored research on *Internet Accessible Mathematical Computation* (IAMC) [18,25,26] and Web-based Mathematics Education (WME) [19] since 1999. ICM led the organization of a series of International Workshops on IAMC [24]. In 2001, the ICM group together with collaborators, began research on a *Web-based Mathematics Education* (WME) framework [19]. Initially, we focused on overall design, computer science, software systems, and content and service interoperability issues. Having addressed, at least initially, many critical technical aspects of WME, we expanded our attention to its practical use, effectiveness, and impact on mathematics learning. Starting in January 2004, Ohio Board of Regents (OBR) Research Challenge funding provided for a pilot project [14] to put WME to an initial trial at Kimpton Middle School, Munroe Falls, Ohio.

The combined efforts of computer scientists, mathematics education experts, and middle school teachers resulted in the rapid prototyping of a WME website with selected contents for 7th grade mathematics and limited in-class trials at Kimpton Middle School.

2.3. What is WME?

WME is a modern distributed system on the Web for mathematics education. The WME system conforms to open standards, works with regular browsers, delivers inte-

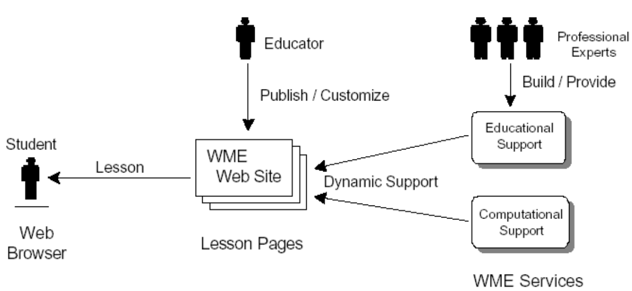

Figure 1: The WME concept

grated and complete lessons, enables easy customization, provides systematic access to client-side and server-side support, and allows these independently developed components to interoperate seamlessly. In short WME seeks to create a *Web for Mathematics Education*. Figure 1 illustrates the WME concept.

Figure 2 illustrates the contents and services WME can combine and integrate for effective Web-based mathematics
education.

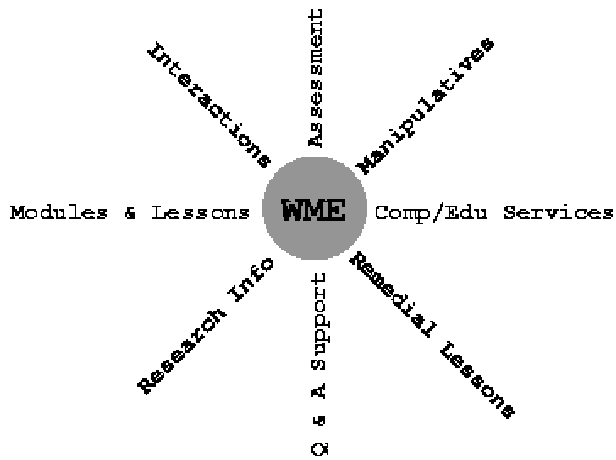

Figure 2: The WME integration

3. WME Architecture and Components

We hope to achieve Web-based mathematical education through an innovative combination of standard technologies and interoperable components.

- Interoperable *Manipulatives* – In-page objects for hands-on experiments designed to support the teaching and learning of specific concepts and skills.
- Interoperable *Topic Lesson Pages* (TLPs) – Complete lessons with identifiable learning goals that integrate motivations, real-life applications, introductions, visuals, examples, hands-on exploration, assessment, guide for teachers and so on.
- Interoperable *Topic Modules* (TMs) – A set of sequenced TLPs to support a specific topic or benchmark within an educational standard such as that of NCTM [30].
- Assessment Support – A well-organized and searchable database of assessment questions at each WME site also supports storing and retrieving of answers and scores. Teachers can import and modify existing questions to construct tests, as well as contribute questions to the database. Databases in different schools, forming a distributed system, mutually reinforce and combine to support the assessment needs at all WME sites.
- Client-side Support – On the user side, common Web browsers can be used. The browser provides support for Standard ECMAScript (Javascript), Document Object Model (DOM), Scalable Vector Graphics (SVG), and preferably other plug-ins.

- Server-side Support – On the Web server side, regular active page and database capabilities support site operation, administration, customization, and configuration [6]. Other services such as MathChat [5] and bulletin boards (Section 7) can also be included.
- Content-markup Support – It is possible to deliver lesson pages in regular XHTML. To provide better document structure for lesson pages, we are investigating the *Mathematics Education Markup Language* (MeML) [20]. MeML supplies *mathematics computation elements and mathematics education elements* that can be used together with regular XHTML and MathML elements for easy content markup. MeML also makes it easy to create interoperable elements and to access WME services that supply various mathematics education capabilities through the Web/Internet. With MeML, a *WME Page Processor* in the form of a browser plug-in (the *MeML Plug-in*) is needed to handle the MeML content [21].
- WME Services – WME compliant Web services can supply a variety of useful functions such as plotting, expression simplification, answer checking, specific mathematical computation, terminology dictionaries, and assessment databases. This way, computations for different areas of mathematics can be supplied to MeML pages from anywhere on the Internet. The WME Page Processor can access WME services through the *Simple Object Access Protocol* (SOAP), the *Mathematics Education Service Protocol* (MESP), and *the Mathematical Computation Protocol* (MCP) [18], just to name a few.

Figure 3 shows the WME system architecture.

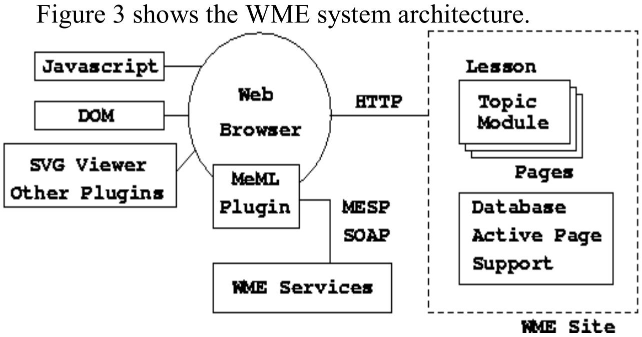

Figure 3: WME architecture

A WME website organizes the mathematics curricula for a particular school. Topic modules and lesson pages are selectable and customizable by individual teachers for different classes. Lesson pages, topic modules, interactive manipulatives, and other lesson elements are interoperable in a plug-and-play fashion among all WME sites.

WME will allow teachers to create or choose from many types of environments for their students. Examples of these are: Specifically designed tutorial programs for assisting students in reviewing mathematical procedures; creating investigations for students to explore and gather data; and interacting with real data to collect, organize, analyze, and explain. In addition, they will be able to modify, change, and augment ready-made lessons with their own ideas and assess students with their own questions.

One of the major obstacles to implementing reform in mathematics education is the lack of an easily accessible context in which to capture learner interest and to challenge students with different mathematical abilities. Research indicates that technology has the capability to help ALL students learn much more significant mathematics than they might otherwise be able to learn without its use [30].

4. A Prototype WME Site for Kimpton

With OBR Research Challenge funding, our group began, in early 2004, a pilot project [34] to study how the WME technologies can best be tailored and applied for actual mathematics education in schools. Details of the pilot project, experiences, feedback, and lessons learned, in the first eight months, have been reported in [14]. In summary, we were very much encouraged by the positive reactions from both students and teachers and proceeded to build a prototype WME site to implement new ideas and improvements, to teach a variety of additional topics, and to performed more in-class trials.

The Kimpton site 7th grade section consists of a number of *topic modules* (TMs) such as *integers*, *percentages*, *proportions*, *length and area*, *number relations*, *fractions*, *probabilities*, and *understanding data*. These are some of the topics in the *Ohio Academic Content Standards* [33] which closely follows the NCTM standards [30].

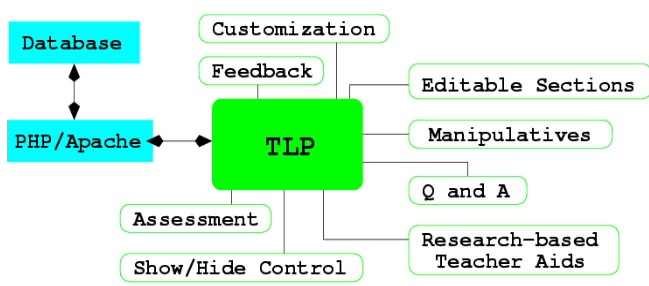

Figure 4: A TLP

Each TM organizes a set of *topic lesson pages* (TLPs) (Figure 4) each focusing on a particular mathematical concept or skill. For example, the *Understanding Data* module contains TLPs ranging from *Statistics in Basketball,* to *bar graphs* and *histograms*.

New TLPs can be added by using existing pages as templates and plugged into TMs by simply listing them in the module index page.

Each TLP generally starts with a short motivation and introduction, followed by hands-on learning supported by specially designed manipulatives integrated into the page.

The interactive activities heighten student interest, focus their attention, and make learning mathematics more engaging. Unlike general-purpose computation tools such as graphing calculators, each manipulative is kept simple, easy to use, and to the point (Figure 5). Thus, they enhance learning and understanding rather than distract students into a different context of a separate tool.

Figure 5: A manipulative

Understanding gained from the interactive activities is reinforced by follow-up questions that can be answered and checked automatically. A TLP usually concludes with additional exploratory or challenge questions to assess and expand the lessons learned.

5. The Ability to Customize

One major advantage of WME is that it supplies classroom-ready TMs, TLPs, and manipulatives prepared by experts. This can be a big help for many teachers.

But the trials at Kimpton also indicated to us that it is critically important for teachers to be able to tailor the existing TMs, TLPs, and manipulatives for individual classes in different schools. For example, a teacher may change a manipulative with a 3x5 grid to one with a 3x7 grid for one class and a 2x2 grid for another by simply editing these values. A meal ordering manipulative will allow different food items and prices to be used.

Many manipulatives are implemented by dynamic HTML based on the standard *Document Object Model* (DOM) and Javascript. Geometry related manipulatives are usually implemented with SVG (Scalable Vector Graphics) code (Section 8). Other interactions are supported by server-side WME services. Efforts are made to keep the manipulatives parameterized for customization and object-oriented for interoperability. We aim to achieve plug-and-play for WME manipulatives, TLPs and TMs.

Also teachers must be able to easily add different types of questions to the pages and collect student answers for later review online. A TLP can become long and confusing for students. Teachers need the ability to selectively show/hide parts of the page to guide student progress and to avoid distractions. Through active pages (PHP/Apache) and database support (MySQL), the pilot WME site offers a password controlled administrative interface for teachers to satisfy these requirements.

The pilot project experience will guide us in building a complete set of WME topic modules and lesson pages to support the entire 7th grade mathematics curriculum, following the NCTM standards [30], and to evaluate its educational effectiveness and impact.

6. WME Model Site

The Kimpton site provides a prototype for the design and implementation of a *WME model site* that can easily deploy WME to different schools. The architecture and organization of the model site makes it easily relocatable and reusable for many grades and schools in the future.

To be easily deployable, a WME model site must capture all the content pages, technological support, and site organization. Figure 6 overviews the model site structure. The homepage identifies the school and links to a class selection page that allows students to choose from

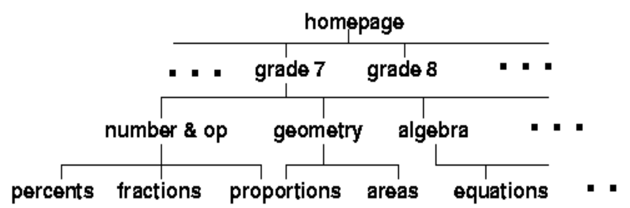

Figure 6: Model site structure

different grades, and their respective course sections and instructors. Each class selection page then includes links to *content standard* areas, whose lesson content is provided by "pluggable" TMs. This is shown in Figure 6, as the *Numbers and Operations* standard is supported by Percents, Fractions, and Proportions Modules.

Given our Web for Mathematics Education, it is within reason to assume that many TMs will be constructed and become openly available for use. But open distribution often implicates the need for customization. Specifically within our research, this requirement was quickly uncovered by the WME pilot project: "...the ability to take the same Web-based lessons and tools and apply them to different classes by different teachers is critical" [14]. Customization, however, is a broad term. We intend to support customization within the following contexts:

- In-School customization – user accounts, grade levels, course listings, course sections.
- In-class customization – TM and TLP selection, management, page content modification, page questions management.
- In-page customization – editing or replacing text, figures, and manipulatives.

Methods of achieving WME page customization and interoperability are reported in a separate paper [7].

7. Online Mathematics Chat and Bulletin Boards

The WME framework presents an online system to aid mathematics educators and students. It then seems practical to deploy chat and bulletin board facilities to students and teachers to promote tutoring help, group discussions, and general interaction unrestricted by classrooms or school hours.

Bulletin boards are Web-based forums used for posing discussion topics, questions, and responses. Because bulletin boards are mostly publicized, anyone in the world can participate at any time, making it an excellent tool for receiving wide-exposure for topics in question. This can be useful on a public or per-school basis where students and teachers can pose questions and answers to each other at their own time. For a mathematics bulletin board (MathBoard), support for mathematics expressions and diagramming must be provided. MathML+MeML seems logical for browser display, but a practical and intuitive mathematics input for users is still under investigation.

Unlike bulletin boards, Web-based chat offers "real-time" interaction, and therefore is only effective when users are connected simultaneously. While chat software are certainly abundant, most naturally aim to provide general socializing features. WME's specifications include server-side support for MathChat [5], a chat protocol that supports MathML and MeML elements for mathematics discussion and on-the-fly WME service invocation (for computations, per se). Because chat communications is live, a question or topic posed expects immediate response. Aside from referencing back logs of connected users there is no latency support within chat frameworks. But because MathChat predestines an MeML page as the product of conversation, these pages are saved on the server and can easily be exported to the WME site for absentees, or perhaps even as a bulletin board discussion topic.

We intend to integrate MathChat and MathBoard with education content by attaching them to individual TLPs, perhaps accessed through the tools menu. The purpose is to make discussions about and references to particular points within each TLP simple and easy.

8. SVG-Based Manipulatives

Geometry and interactive diagrams are important in mathematics education. Scalable Vector Graphics (SVG), a W3C standard, is an XML application that supports vector graphics and animation. Being a Web standard, SVG is expected to soon enjoy native browser support, as a graphics format as well as an XML application.

With Javascript and SVGDOM, geometry-oriented interactive manipulatives can be achieved and delivered to end users efficiently. Such manipulatives are small, fast to load, and run on the client-side. Figure 7 shows such a manipulative.

We are also building a system to make creating interactive diagrams and dynamic geometry learning easy. This includes an SVG library a Web-based authoring tool GeoSVG, and a GeoSite website to support publishing manipulatives authored using GeoSVG. GeoSVG and GeoSite together deliver a modern, open, and effective tool for plane geometry learning and interactive diagram creation [13].

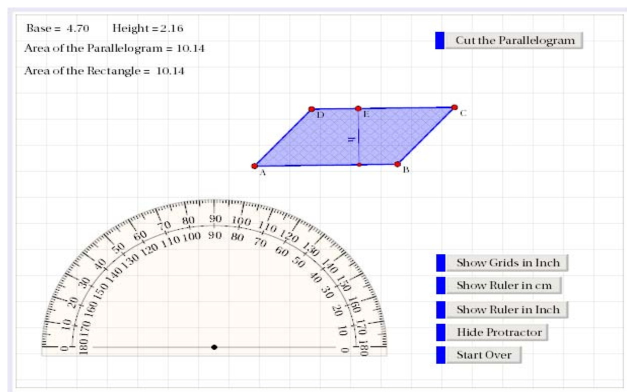

Figure 7: An SVG-coded manipulative

With source code in XML, SVG graphics can be generated, processed and transformed on the server side and client side with relative ease. For WME manipulatives that involve animation with sound and video, however, Flash is still useful.

9. The Assessment Database

Teachers need to know the progress students are making and how well each student is doing. WME can help teachers do assessments and we are building a Distributed Mathematics Assessment Database (DMAD) for WME.

DMAD consists of *local databases* located at individual WME websites. A local database collects assessment questions contributed by local teachers to be shared with others. Teachers can search and import assessment questions, edit and revise them to use in tests and assignments. They can also author new questions and contribute them to DMAD to be shared locally or widely.

DMAD supports true-false, multiple-choice, short-answer, and essay questions. Questions may contain text, pictures, graphs, and formulas. DMAD provides online tests, automatic grading of student answers, real-time feedback, and statistics of student performance. Questions in a test can be displayed in various permutations automatically to enable test taking by students at close proximity, for example.

When authoring assessment questions, a teacher can indicate the correct answer as well as connect incorrect answers to common misconceptions or missing background knowledge. DMAD can help correlate such diagnostic information with TLPs which, in turn, can be used to help students overcome difficulties exposed by assessments.

The initial design of DMAD [1] has laid a good foundation for further work. Use of DMAD by teachers in actual classes will lead to refinements and wider distribution of this useful feature of WME.

10. WME Advantages

Our experience with WME has provided indications that it can bring these advantages.

- *Accessibility* – Is accessible 24/7 by students and teachers with regular Web browsers, usable in classrooms, in computer labs, at home and on the road; makes mathematics education content widely accessible in and out of classrooms and schools.
- *Compatibility and Interoperability* – Follows modern Web standards and compatible with all XML tools; participates in and reinforces the "technology in schools" movement in the USA; is easy to share, extend, and internationalize; allows easy incorporation of new Web and Internet tools and technology; provides interoperable contents and services.
- *Richness and Variety* – Empowers school teachers, independent educators, mathematics experts, and computer professionals to contribute to mathematics education as an individual endeavor; forms a potentially un-limited set of mathematics education contents, services and manipulatives provided by experts; shares widely contents and services for mathematics education at all grade levels; includes educational components ranging from lessons to interactive multimedia presentations, from tests to homework, from targeted manipulatives to powerful general-purpose computation tools.
- *Integrated, Dynamic, and Classroom Ready* – Integrates text, graphics, interactions, explorations, and assessment in concept-centered lesson pages that are generated dynamically; provides page modifyca-tion, question posing, and answer collecting capa-bilities inside individual lessons; allows teachers to import, select, customize as well as mix-and-match ready-made contents for individual classes; supplies easily customizable model WME sites to schools.
- *Efficient Communication* – Maintains a flexible and effective channel of online communication between teachers and students; supplies math-capable chat and message boards; provides much-needed opportunities for written communication by students; enables teachers to interact with each student in the entire class at once, and allows students to give answers privately without interference.
- *Concepts not Steps* – Enables students to visualize and explore their own theories, conjectures, and ideas through concept-specific manipulatives; supports automatic plotting and diagramming; accessing mathematical algorithms as on-Web tools; frees teachers and students from tedious and time consuming work such as long calculations, plotting, and managing physical manipulatives so they may focus on concepts and problem-solving strategies; easily enables hands-on practice for computational skills when desired.
- *Educator Support, Convenience, and Control* – Allows teachers to control who may access the Web-based contents; provides real-time control of the visibility of parts of lesson pages to better focus student attention; helps teachers organize questions, answers, tests and grades; provides assessment databases with modifiable questions; enables teachers to pose their own questions to assess each and every individual student in the class at once.
- *Real-world Motivations* – Makes it easy to put up good real-world examples to motivate learning of mathematics; allows easy access to interesting and current examples of real applications on the Web.
- *Practical and Flexible* – Is readily available to teachers and students in classes with Web access; delivers complete lessons that can be free; easy to use as students and teachers are mostly already familiar with personal computers and Web browsers; easy to update and improve; has good potential for commercialization.
- *Interactive, Hands-on and Self-paced* – Enables students to explore concepts/ideas, make progress, and answer questions at their own pace, individually and privately, or in small groups; provides task-specific hands-on experiments through animation, interaction, exploration, making mathematics more exciting, less abstract, and easier to understand; allows students to review materials at anytime, to easily obtain help from parents and siblings, or to move ahead and explore more extensively.

11. Acknowledgements

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