Support for Online Mathematics Education: MeML and WME Services

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Abstract

As the Web-based Mathematics Education (WME) system develops and undergoes in-class trials, we are investigating better technologies for page markup and support services. The Mathematics Education Markup Language (MeML) is an XML application specifically designed to structure WME lesson pages and increase element interoperability. A client-side plug-in, Woodpecker, supports the display and operations of MeML pages. The Mathematics Education Service Protocol (MESP) defines a uniform interface for accessing, controlling, deploying, and customizing WME services. MeML and MESP are important for WME to become a "Web for Mathematics Education" based on interoperability of components and expertly constructed pages and services that are customizable.

1. Background

Web-base Mathematics Education (WME) [6] is an on-going project at the Institute for Computational Mathematics (ICM/Kent State University). The goal of WME is to develop a modern Web and Internet based system for supporting, enhancing, and delivering mathematics education at all levels. [3, 7]

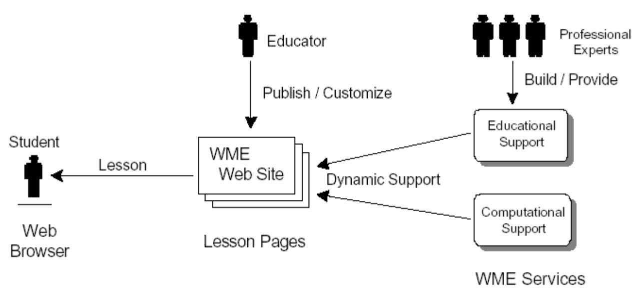

Figure 1: WME concept

Through an innovative application and combination of open Web technologies, WME aims to create a Web for mathematics education [7] where education content and support services are interoperable on the Web and can be created, deployed and maintained on a distributed basis (Figure 1). Unlike stand-alone manipulatives [14] or lesson plans on the Web, a WME website is organized to systematically supplement complete mathematics curricula in any given school. WME can create an effective and wide-reaching system to facilitate the interaction and cooperation among all potential contributors to and beneficiaries of mathematics education.

A pilot project puts WME to in-class trial at Kimpton Middle School (Munroe Falls, Ohio). The feedback from the 7th-grade mathematics teachers and students was overwhelmingly positive and very encouraging [1]. Close cooperation among computer scientists, mathematicians, mathematics education experts, and school teachers is adding content, making improvements, and improving features to WME while more in-class trials are continuing.

With WME, a school gets its own WME website consisting of a set of topic modules (TM) and server-side support. Each TM contains a number of topic lesson pages (TLP) [3] that are complete and classroom-ready. Typically, each TLP helps teach some particular mathematical concept or skill with motivation, introduction, interactive activities, and teacher-guided hands-on experimentations, as well as chances to answer probing questions and getting the answers checked automatically or later by the teacher.

TMs and TLPs are plug-and-play components for all WME sites. They are also fully customizable and can easily be modified by individual teachers to suit their classes. Concept-specific manipulatives, supplied by experts and modifiable by teachers and students, are pre-installed to supply just the right interaction and exploration to add interest, enhance learning, and increase understanding. Teachers can easily control lesson pages, pose questions, and provide feedback to students, as well as monitor and assess student progress.

2. Introduction

The pilot WME site uses current Web technologies including HTTP (Apache), XHTML, MathML, [10] ECMAscript, the Document Object Model (DOM), active pages (PHP), and database support (MySQL). As such technologies get WME started and growing, the usage experiences and in-class feedback increasingly point to the desirability of improved technological support for WME, particularly in the areas of page markup and service access.

A cornerstone of WME is the TLP which is obviously not a general Web page. Therefore, it is hard to use XHTML and MathML to capture the structure and organization of such mathematics education pages. The design and implementation of MeML, an XML [9]

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application, to serve as the markup language of WME pages is presented. Our group has been investigating MeML for some time [2] and as the WME system grows and evolves, it becomes increasingly clear that a reexamination and redefinition of MeML is indeed necessary.

MeML elements (markup tags) can be used together with MathML, XHTML, and other XML-compatible languages to structure a TLP. MeML elements are designed to capture the macro structures inside a TLP and create interoperable page elements, making them self-contained and easily interchangeable among TLPs. Such an organization can also enhance interactions and communications among these page elements.

The better support afforded by MeML for mathematics education concepts and terminologies also means easier authoring by developers and teachers. An authoring tool built upon MeML can provide an even easier visual interface for school teachers to modify existing TLPs as well as authoring new ones from supplied templates.

Also critical to WME is the use of in-page interactive manipulatives that are combined and integrated with education contents to illustrate and teach specific mathematics concepts and skills. The hands-on experiences are very much appreciated by students and tend to keep them focused on the educational pages. Manipulatives in the current WME pilot site are implemented in a number of ways. Client-side manipulatives are usually in ECMAscript or Scalable Vector Graphics (SVG) and server-side manipulatives rely on PHP and CGI. The manipulative-to-page and manipulative-to-manipulative relations and interfaces are currently treated on an ad hoc basis. In addition to manipulatives, a TLP may also involve WME services such as terminology look-up and assessment support, just to name a few. In general, a WME service is any dynamic functionality, other than page generation, supplied to a lesson page. With the WME pilot project, the number and variety of services have grown to a point where a systematic way to provide WME services ought to be considered.

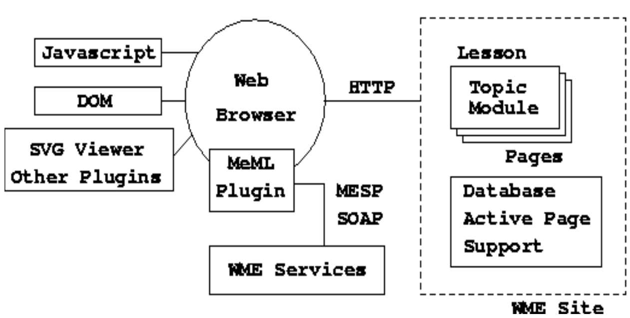

Figure 2: WME architecture

While MeML can help organize the markup of WME services and manipulatives inside a TLP, we still need to consider a uniform interface for deploying, invoking, interacting, and customizing them. A significant difference between WME and other online educational infrastructures is its interoperability. WME modules, lesson pages, manipulatives, and services are interoperable. Making WME services interoperable through the Mathematics Education Service Protocol (MESP) is also described. Figure 2 illustrates the relation of MESP to the WME system.

3. Mathematics Education Markup Language

MeML is defined by XML. It aims to provide effective and expressive means for structuring and delivering mathematics education content on the Web. Its definition is a result by examining current textbook and online math materials, testing mathematics courseware, consulting education experts, following NCTM principles [15], and obtaining experience from RAD prototype.

MeML defines about 54 elements, which are grouped into five categories. They are: *content elements*, *education elements*, *organization elements*, *computation elements*, and *system elements*. Table 1 lists all categories as well as elements of each category.

Table 1: MeML elements

Category	Elements
Content	*concept, skill, terminology, problem, identity, theorem, formula, diagram, equation, algorithm, definition, property, operation, notation*
Education	*lesson, example, exercise, assessment, homework, test, remediation, question, diagnosis, answer, solution*
Organization	*abstract, syllabus, roster, guide, hint, chapter, summary*
Computation	*variable, expression, computation, mathgraph, assign, range, with, condition, function*
System	*certifieduser, monitor, editable, userinput, interaction, use, parameter, wmeservice, configure, sensitive, manipulative, deploy, vsection*

Each category supports a special aspect of educational work. They facilitate authoring, delivering, storing, processing, and exchanging mathematics education materials over Web.

- **Content elements** – each content element can be used to define a knowledge unit. Their definitions provide an effective way to store mathematics knowledge in XML form. Teachers can also use them to construct complicated concept, like definition of Set as well as related operations and properties of Set. The knowledge stored in XML hierarchical structure also makes it easy for online search engine to find education material over Web. Content element itself supports cross referencing to ease online knowledge finding.

- **Education elements** – education elements are used to describe the educating activities happening in and out of the physical classroom. They have dual roles in MeML. Somewhat like content elements, education elements support structural storage of education materials. On the other hand, they help teachers to move their educating activities onto the Web. For example, tag "<homework>" helps teacher to assign homework to students. It may include several questions which may be either given by teacher manually or served dynamically by backend WME services. Moreover, with support of WME services, teachers can enjoy the convenience of automated grading and student performance analysis.
- **Organization elements** – Organization elements are used to help teachers prepare their online curriculum. Their "reference" attribute helps teachers to easily link various education materials, which may be distribute anywhere on the Web, into their own online lectures. These elements, by nature, support online course finding.
- **Computation elements** – mathematics education must be ready for any type of online computation. First, MeML tag "<wmeservice>" can associate each MeML page with one or more compute engines such as Maxima or Maple. [2] Then computation elements help teacher to prepare mathematics expressions, control computing sequences, submit computation transactions, and update page content using returned result. With the help of other MeML tags, like system elements, it is easy for teachers to give an interactive learning environment to students.
- **System elements** – Formally known as interaction element, system elements control the runtime behavior of MeML page in an understandable way to math teachers. For example, tag "<certifieduser>" provides authentication service to MeML pages, which can check the identity of page visitor and control the access to the content in MeML page. Some other system elements, like tag "<editable>" and tag "<monitor>" control the display of MeML page according to the information provided by "<certifieduser>" at runtime.

All of these MeML elements in a whole cover educating works from note/textbook preparation to face-to-face in-class activities and out-class assignments. WME services will bind the support from computer specialist to students and teachers on every aspect under the control of WME Framework.

4. WME Services

Computer experts worldwide can develop any WME service they want. As long as they follow the rules of WME Framework and make their service conforming MESP interface, the dynamic content generated by their services could be delivered to students' and teachers' Internet browser automatically.

MESP requires each WME service to provide a XML-based description about its content. So, teachers can browse, test, and choose appropriate WME services at early stage of creating work. Figure 3 presents a working diagram of using WME service in WME Framework.

Another valuable property of MESP is that it can deliver part of configuration of a MeML page, which is generated by tags <wmeservice>" and "<configure>", to the WME servers it is connecting. This opens the door for WME services that are serving current MeML page to cooperate with each other to provide more complicate and powerful services through MESP.

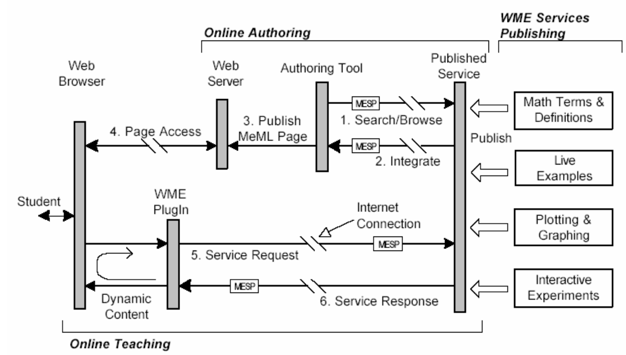

Figure 3: Accessing WME services

To facilitate the delivering of WME technology to math teachers and students, WME Framework defines some core services to incorporate with MeML. They are: *account service*, *repository service*, *computation service*, and *terminology service*.

Account service helps author of MeML page to validate and identify page visitor, assign virtual passport to visitors which can be used with any WME server, and deal with the relationship among student, teacher, and course.

Repository service provides general storing and retrieving service to dynamic data that is generated from educating activities. Each piece of data is associated with MeML page URL, name and ID of MeML tag that creates the data, and virtual passport of current page visitor. The explanation to the meaning of data depends on the corresponding MeML tags. With repository service, it becomes possible to collect students' performance data in a unified format. Thus, an arbitrary WME service, like online performance analysis, can utilize the data without prior knowledge of how teachers prepare course pages.

Computation service encapsulates online mathematics computation served by powerful compute engines like

Maple, Maxima, etc. Generally, several compute engines can be accessed through one IAMC server [4, 5, 11] which hides the difference of engines' programming interfaces.

Terminology service supplies formal and in-depth explanation to mathematics knowledge defined by content elements. It is rather flexible and there are many existing Internet resource can be used as terminology service directly, for instance, the MathWorld of Wolfram Research Inc.

Table 2 shows part of interface description of WME account service that is serving Kimpton Middle School. Table 3 shows the returned result of a call to ASSOCIATE service, which helps teacher to find all students who register same course or vice versa. Note that the XML tags that appear in Table 2 and 3 are MESP, and not MeML.

Table 2: WME account service

```
<?xml version="1.0" ?>
<wmeservice name="ACCOUNT" wmeurl="http://
   wme.cs.kent.edu/wme/service/account.wme">

<bind protocol="mesp" />

<service name="LOGIN">
<parameter name="user" type="string" />
<parameter name="password" type="string" />
<description>Create a valid WME Passport for
WME users</description>
</service>

<service name="VERIFY">
<parameter name="user" type="string" />
<parameter name="password" type="string" />
<description>Check if a client browser has a
valid WME Passport </description>
</service>

<service name="MONITOR">
<parameter name="passport" type="string" />
<parameter name="course" type="string" />
<description>Given a valid user (Student)
Passport and a course number, check if there
is an associated monitor (Teacher) is in
active state (Teacher has login somewhere in
system. Her assigned Passport is not expired
right now and she did not logout.)
</description>
</service>

<service name="ASSOCIATE">
<parameter name="passport" type="string" />
<parameter name="course" type="string" />
<description>Get Teacher's Account ID that
is associated with the course chosen by
Student or vice versa.</description>
</service>

</wmeservice>
```

Table 3: Calling ASSOCIATE service

```
<?xml version="1.0" ?>
<mesp packet="RESPONSE">
  <service name="ASSOCIATE">
  <response state="SUCCESS">
    <associate>
      <course>Calculus</course>
      <caller type="T">Kim</caller>
      <callee>Student_A</callee>
      <callee>Student_B</callee>
    </associate>
  </response>
</service>
</mesp>
```

The definition of WME core services presents not only the support to MeML, but also our consideration to commercial organization of online education in the future. Simply put, these WME services are not restricted to a single proprietor. For example, an institute can setup an online virtual school by running an account service only. They could create course pages to utilize the computation service provided by ICM/Kent State University, and provide an interactive Mathematics Lab to their students without worrying about any complicate technologies for advanced computation. They could also use the terminology service for free from the MathWorld. Similarly, they can utilize abundant WME services distributed over Internet as long as they use WME Framework as their education infrastructure.

5. Woodpecker

An MeML page may contain MeML, XHTML, and MathML element. As a new XML language, MeML tags need to be acceptable by Web browser. We develop a MeML processor prototype - Woodpecker [2,8], which runs as a plug-in of Web browser.

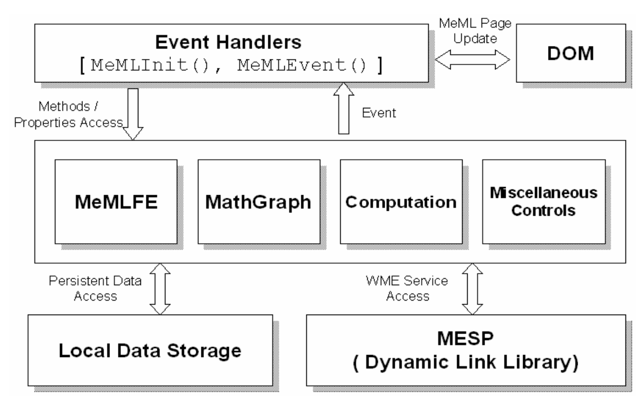

Figure 5: MeML plug-in prototype - Woodpecker

With Woodpecker, a MeML page is translated into XHTML/MathML, JavaScript, and references to MeML plug-ins using XSLT processors. Then, the translated page is submitted to the Web browser for display and interaction.

When the Web browser loads the translated page, references to MeML plug-ins will be instantiated as in-page objects. Depending on the type of browser, the object instances may take the form of ActiveX Controls (in Internet Explorer) or Plug-Ins (in Netscape Navigator). Some important MeML plug-ins are MeMLFE, MathGraph, and MathComputation. They could be visible or hidden in rendered page. When user interacts with the page, a series of in-page events may be generated from either XHTML components or MeML plug-ins. The ECMAScript event handlers treat these events by interfacing to appropriate MeML plug-ins to support the functionality of specific MeML elements. The design of Woodpecker is shown in figure 5.

6. MeML Live Examples

For better understanding of MeML language and WME technologies, following live examples are given to show the power of MeML.

6.1. Import/Reuse Manipulatives

Table 4: Import manipulative

```
<p>To start over simply <a href="">reload
the page</a>.<br /></p>

<manipulative id="pizza" rid="pizzapan"
type="import" wmeurl="http://wme.cs.kent.edu
/kimpton/topicobjects.meml"
scriptlib="http:// wme.cs.kent.edu/kimpton
/javascripts/grid5.js"
csslib="http://wme.cs.kent.edu/kimpton
/pizzanb.css" />
```

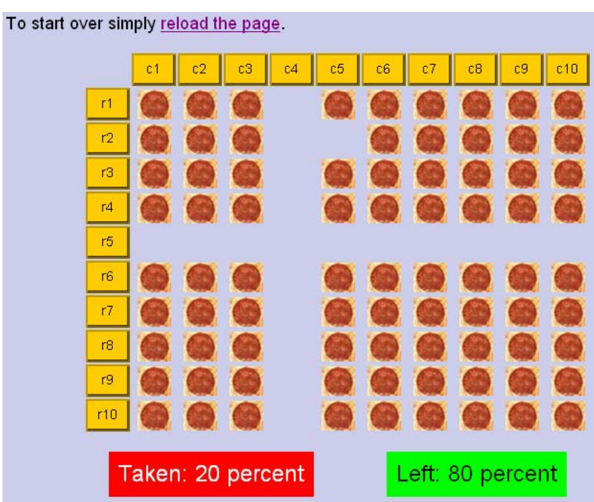

Figure 6: MeML page of manipulative

'Manipulative' means a self-contained, customizable, and interactive MeML content block which can be import/export between different MeML pages. Above example is a typical manipulative that serves a virtual pizza to K-7 student. Student can play with it by taking off or putting back pizza. This manipulative will calculate the percent left and taken and show it to student. The point is that the manipulative is actually in another MeML page. But present page can reuse it using "<manipulative>" tag with "type" attribute set to "import". This example doesn't use any WME service. The actual display of MeML is shown in figure 6.

6.2. Customize Content and Control Access-Time

Table 5: Using system elements

```
<certifieduser wmeurl="http://localhost/WME
/service/account.wme">

<monitor id="mon1" timeout="5">
  <h3>Discussion</h3><ul><li>

  <vsection id="vsec2">
    <p>If 50% of the cost of your meal
actually goes to pay labor at the
restaurant, what is the labor cost for your
meal?(Type just a number, without any dollar
sign.)
    <deploy id="first" rid="calculate"
            manipulative="diningout">
    <parameter name="#input" value="labor"/>
    <parameter name="#percent" value="0.5"/>
    </deploy></p>
  </vsection>

  </li><li><p>If you wish to leave 10% (ten
percent) tip based on the total before tax,
how much is your tip?
    <deploy id="second" rid="calculate"
              manipulative="diningout">
    <parameter name="#input" value="tip"/>
    <parameter name="#percent" value="0.1"/>
    </deploy>
  </p></li>
  <!-- ... ...  -->
</monitor>
```

In this example, tag "<vsection>" defines a view section that teacher can choose to keep or remove for teaching purpose. Tag "<monitor>" defines a content block which can only be accessed under teacher's monitor. The block is invisible in student's browser before teacher click "Start Monitor" button or after teacher click 'End Monitor' button in teacher's browser. Figure 7 shows the MeML page displayed in teacher's browser. Figure 8 shows the display of same MeML page in student's browser. As we see, all of control information is absent in student view. This example requires the support of WME account service and repository service. The change, that teacher makes to page, will update student browser quickly without modifying original MeML page because of repository service. The account service is required because several teachers may use same MeML page. They should not distort each other's courses. Students can only see the changes made by their own teachers.

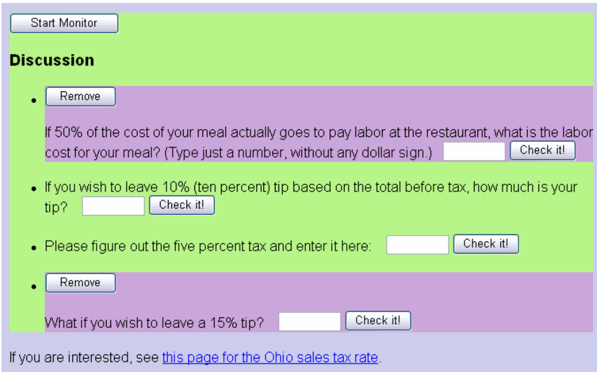

Figure 7: MeML page viewed by teacher

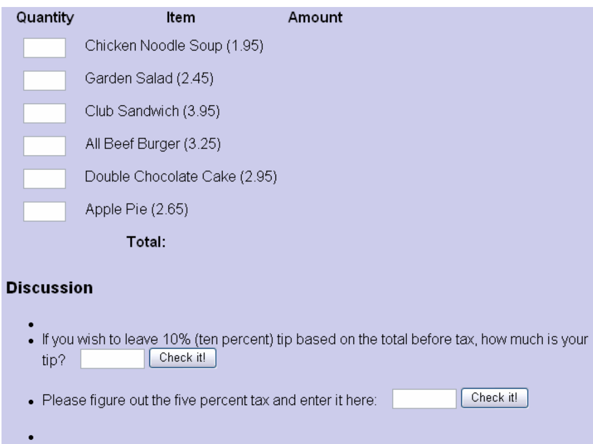

Figure 8: MeML page viewed by student

6.3. Interactive Mathematics Computation

Table 6: Using computation element

```
<variable id="var1" type="symbol"
          range="range1">x</variable>
<variable id="var2" type="symbol"
          range="range1">y</variable>
<variable id="from" type="real"
          value="-10.0">a </variable>
<variable id="to" type="real"
          value="10.0">b </variable>
<range id="range1">
  <use name="from"/><use name="to"/>
</range>

<expression id="func" encoding="Infix">
     x^2*Cos[y]- Sin[x]*y^2</expression>
<diagram title="Diagram 1" id="sincurve">
  <mathgraph id="plot"
             wmeurl="mcp://wme.cs.kent.edu"
             width="400" height="300">
   <parameter name="operation">
       Plot3D</parameter>
    <parameter name="function">
       <use name="func" /></parameter>
    <parameter name="variable">
       <use name="var1" /></parameter>
    <parameter name="variable1">
       <use name="var2" /></parameter>
  </mathgraph>
</diagram>

<interaction target="plot">
  <p><table><tr><td>Expression: </td><td>
  <userinput type="expression" name="func"
             encoding="Infix" />
  </td></tr><tr><td>1st Variable: </td><td>
  <userinput type="symbol" name="var1" />
  </td></tr><tr><td>2nd Variable: </td><td>
  <userinput type="symbol" name="var2" />
  </td></tr><tr><td>Minimal Range: </td><td>
  <userinput type="real" name="from" />
  </td></tr><tr><td>Maximal Range: </td><td>
  <userinput type="real" name="to" /></td>
  </tr></table></p>
</interaction>
```

This example uses computation elements "<variable>, <expression>, <mathgraph>, <parameter>", and system elements "<interaction>, <userinput>, <use>" to create a mathematics graph with the help of computation service. It uses interaction elements to associate the user's input with the mathematics entity defined by computation elements. The resulting page (figure 9) shows an interactive tool that help student understand the mathematics meaning of curve and plane of any function with two variables.

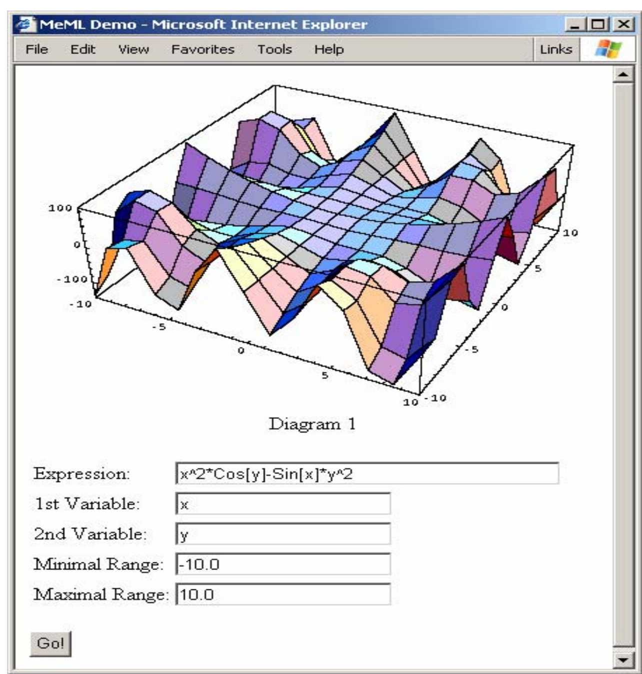

Figure 9: Interactive mathematics computation

7. Conclusion

MeML is specially designed for math teachers. It is purely education-oriented. It supports structural storage of education materials, dynamic and interoperable content creation, and online mathematics computation. WME service and MESP provide a uniform interface to link all kinds of education resources into MeML page. Woodpecker, as a plug-in module, implements all the functions of MeML language in regular Internet browser. These technologies can greatly facilitate the creating work of education pages and online courseware, support interoperation and customization of Internet education resource, and promote the online delivery of math education. They still follow modern Web standard and keep maximum compatibility with current Web technologies.

8. Future Work

WME Framework is an ambitious vision and there is much work ahead. As the core of WME Framework, the definition of MeML and implementation of Woodpecker and WME Services speed up our progress to deliver WME technology to math teachers and online learners. To make WME services more interoperable and easily accessible on the Web, we are defining Mathematics Education Service Protocol and creating support libraries for it. The software components of WME Framework will continue to be tested and refined. The work of complementary subprojects will also be merged into WME Framework step by step. A MeML authoring tool, MadMath, is being designed to make authoring task of online math education pages in MeML much easier.

9. Acknowledgements

Work reported herein has been supported in part by the National Science Foundation under Grant CCR-0201772 and in part by an Ohio Board of Regents Research Challenge Grant. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of any funding agency.

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